the Hopfield network

Hopfield 1982
Grossberg 1983, earlier?

Hopfield network

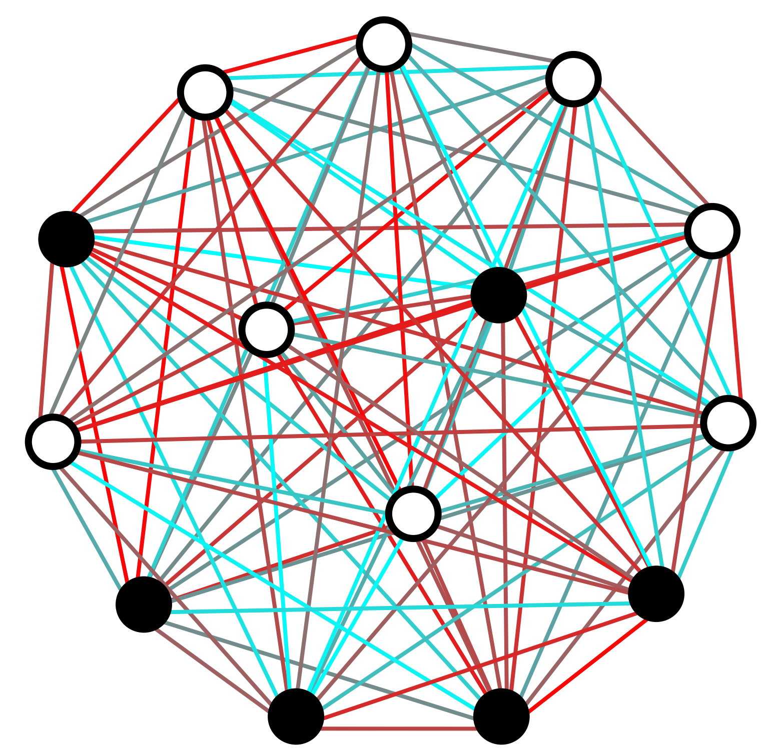
Associative memory:
Recall whole pattern associated with partial information

Recurrent network with no explicit readout.
Internal neural activity is the target.

N binary 'neurons': $s_i = \pm 1$

p input patterns: ${\xi_i}^{\mu} = \pm 1$

N^2 synapses: $J_{ij} = J_{ji}\ ,\ J_{ii}{=}0$



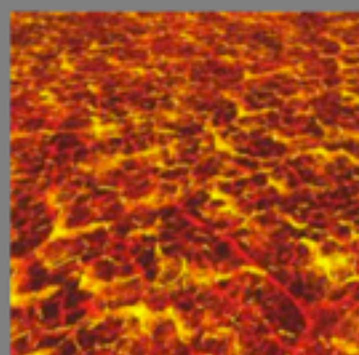
Associative memory





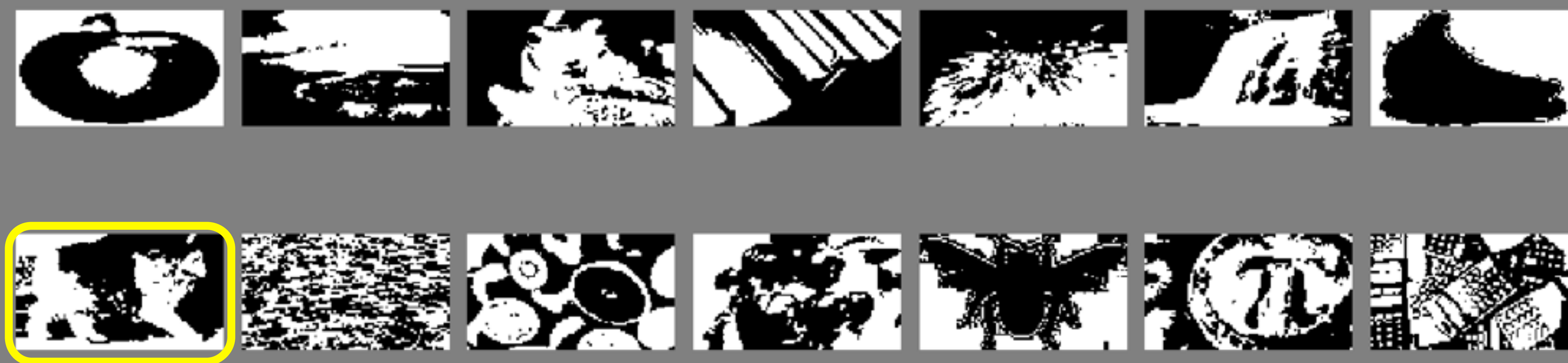









Associative memory



Corrupted pattern

Retrieved pattern

Hopfield network

Dynamics: $$s_i \to \mathrm{sgn} \sum_j J_{ij} s_j$$

Update each neural activity to better match patterns stored in the synapses J_{ij}

Learning rule: $$J_{ij} \to J_{ij} + \xi_i^\mu \xi_j^\mu$$

i,j : neuron index
μ : pattern index

Update synapses to better match target patterns ξ

Hopfield network

Dynamics: $$s_i \to \text{sgn} \sum_j J_{ij} s_j$$

Update each neural activity to better match patterns stored in the synapses J_{ij}

Learning rule: $$J_{ij} = \sum_\mu \xi_i^\mu \xi_j^\mu$$

i,j : neuron index
μ : pattern index

Update synapses to better match target patterns ξ

Hopfield network

Dynamics: $$s_i \to \operatorname{sgn} \sum_j J_{ij} s_j = \operatorname{sgn} \sum_{\mu,j} \xi_i^\mu \xi_j^\mu s_j$$

$$= \operatorname{sgn} \sum_\mu \xi_i^\mu \left(\sum_j \xi_j^\mu s_j \right)$$

$$= \operatorname{sgn} \sum_\mu \xi_i^\mu v^\mu$$

Learning rule: $$J_{ij} = \sum_\mu \xi_i^\mu \xi_j^\mu$$

Hopfield network

Dynamics: $$s_i \to \operatorname{sgn} \sum_j J_{ij} s_j = \operatorname{sgn} \sum_{\mu,j} \xi_i^\mu \xi_j^\mu s_j$$

$$= \operatorname{sgn} \sum_\mu \xi_i^\mu \left(\sum\nolimits_j \xi_j^\mu s_j \right)$$

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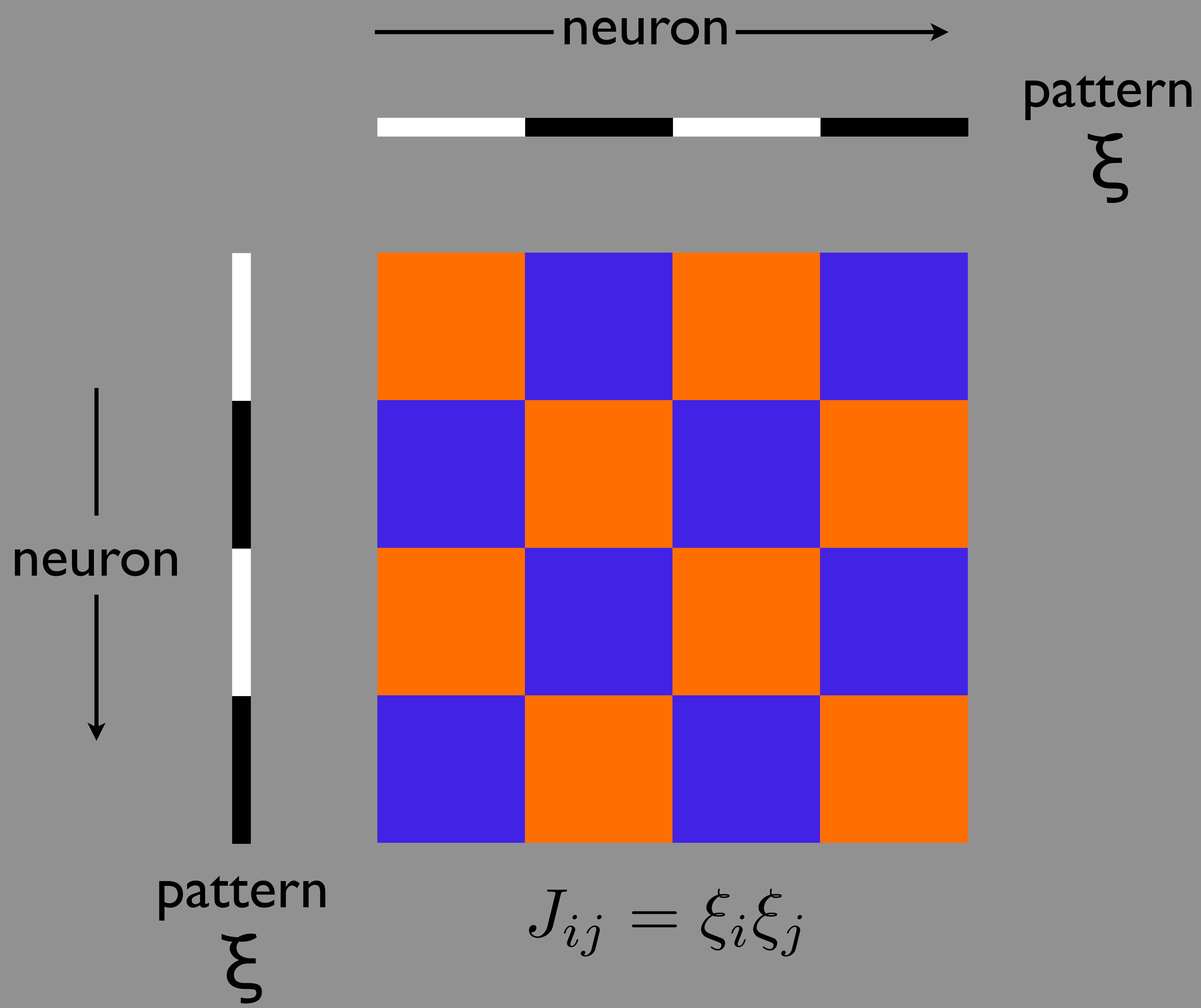
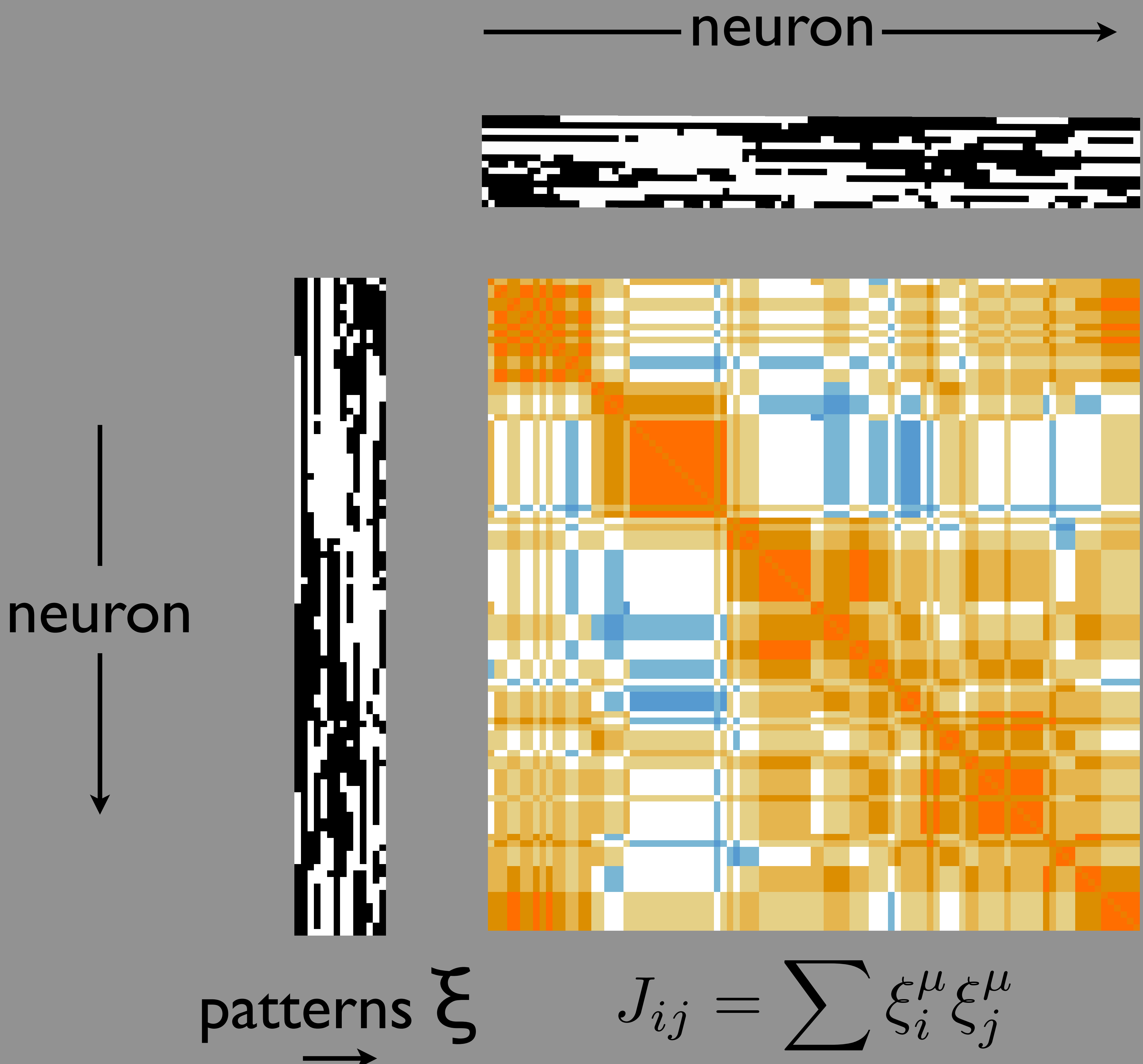
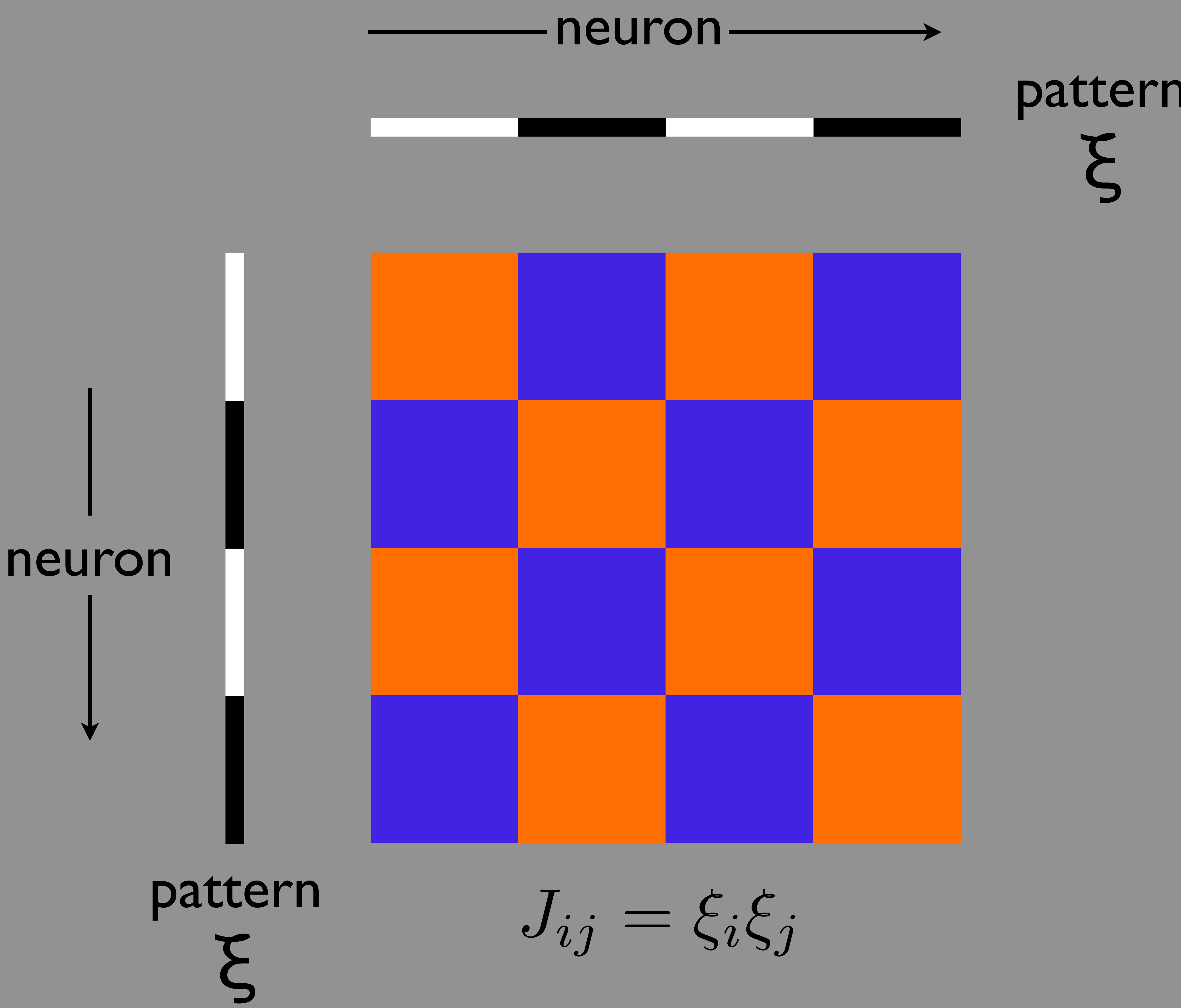
Activate neuron i based on votes from each pattern, weighted by similarity to current state

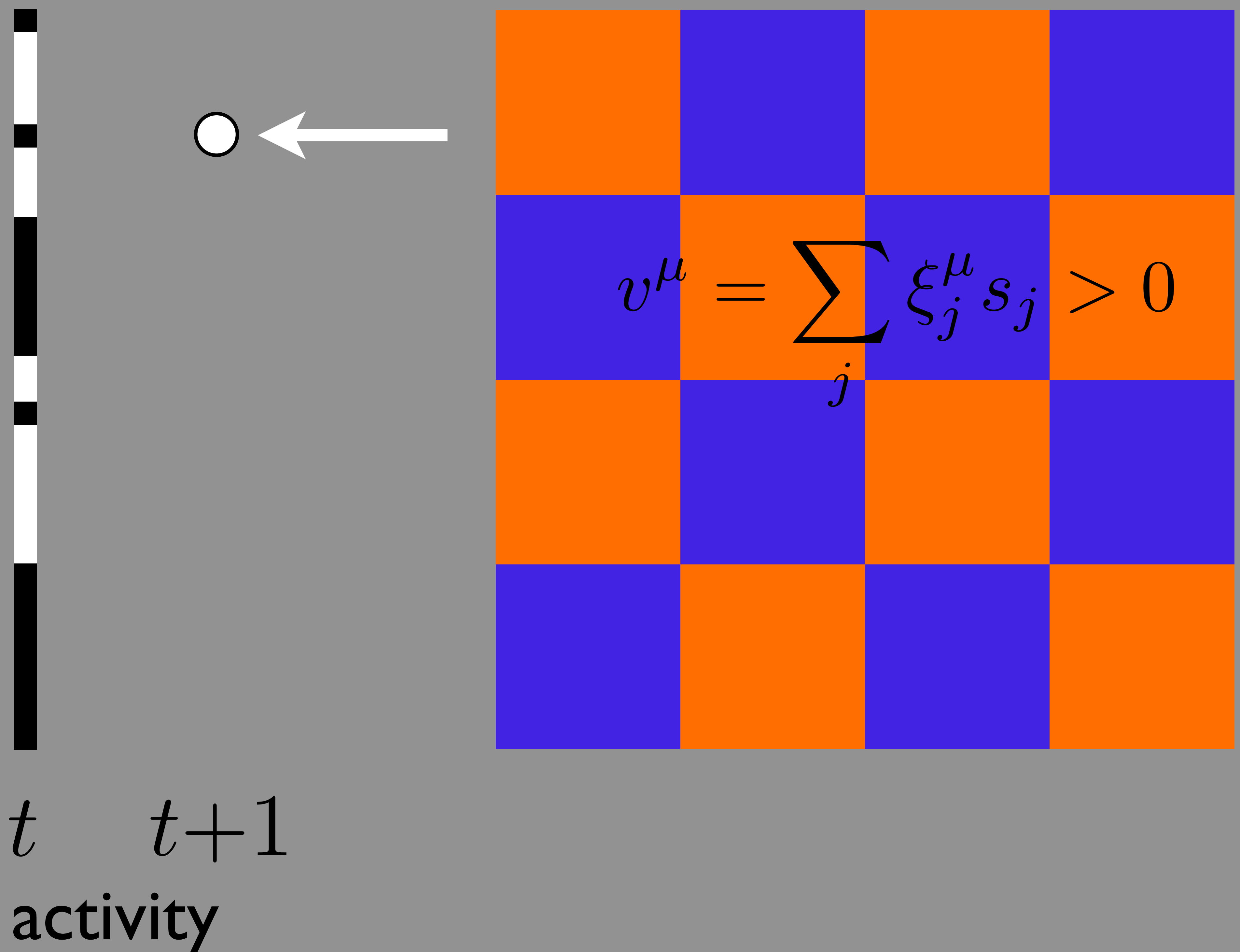

neuron
pattern
ξ
neuron
pattern
ξ
$J_{ij} = \xi_i \xi_j$



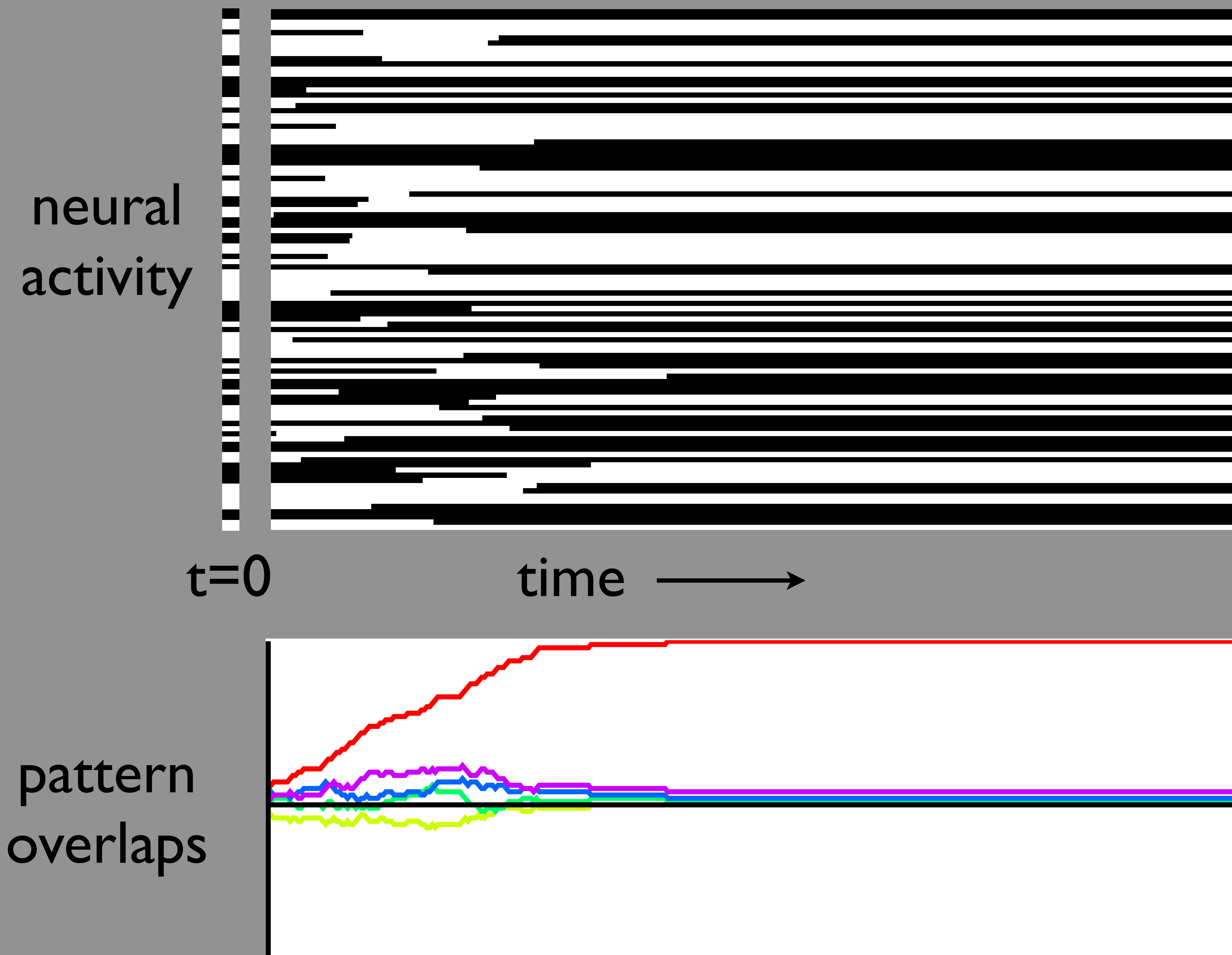
$$J_{ij} = \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$$


neuron
pattern
ξ
neuron
pattern
ξ
$J_{ij} = \xi_i \xi_j$

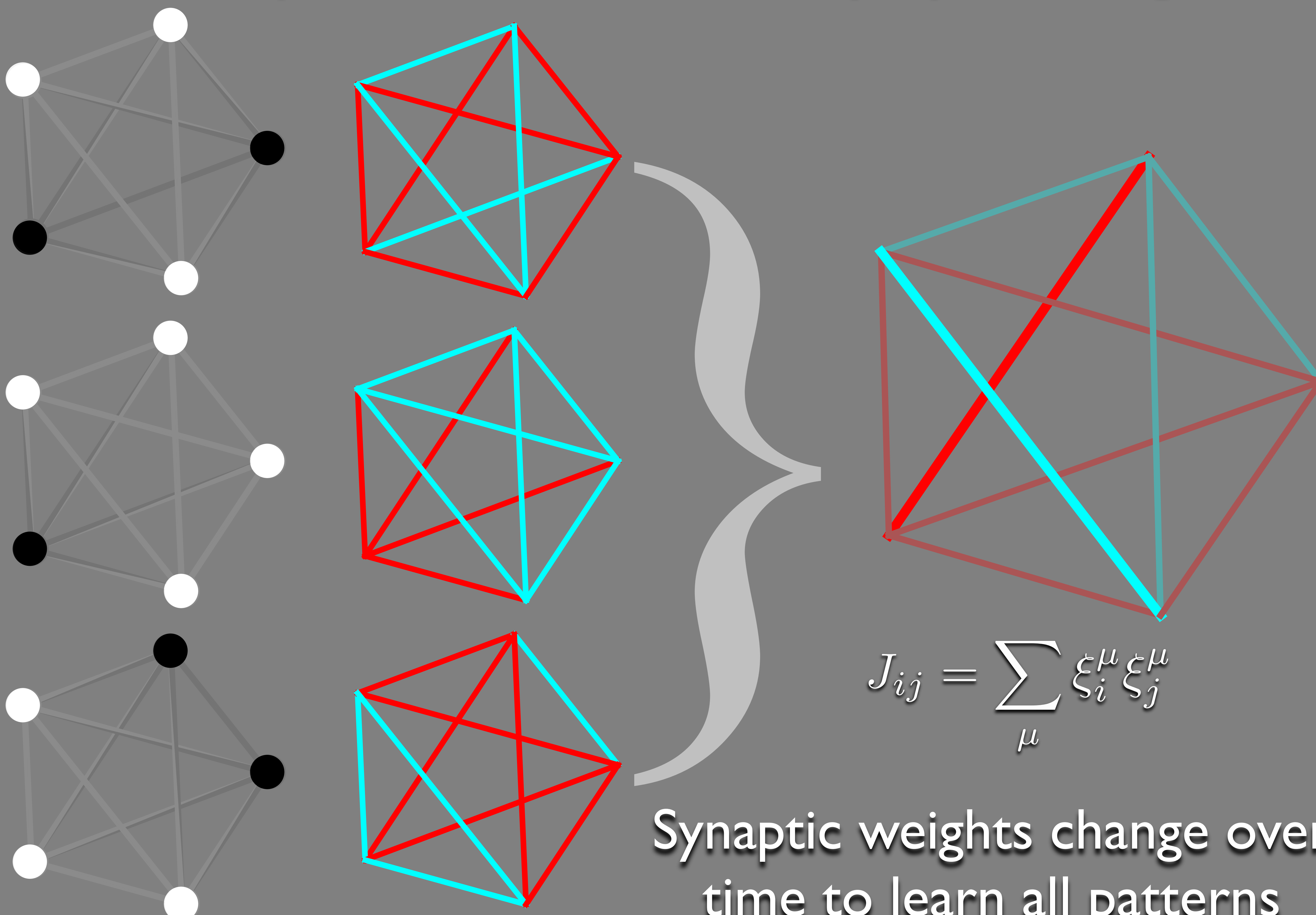
Implementing dynamics $s_i \to \mathrm{sgn} \sum_j J_{ij} s_j$



Visualizing dynamics of Hopfield network

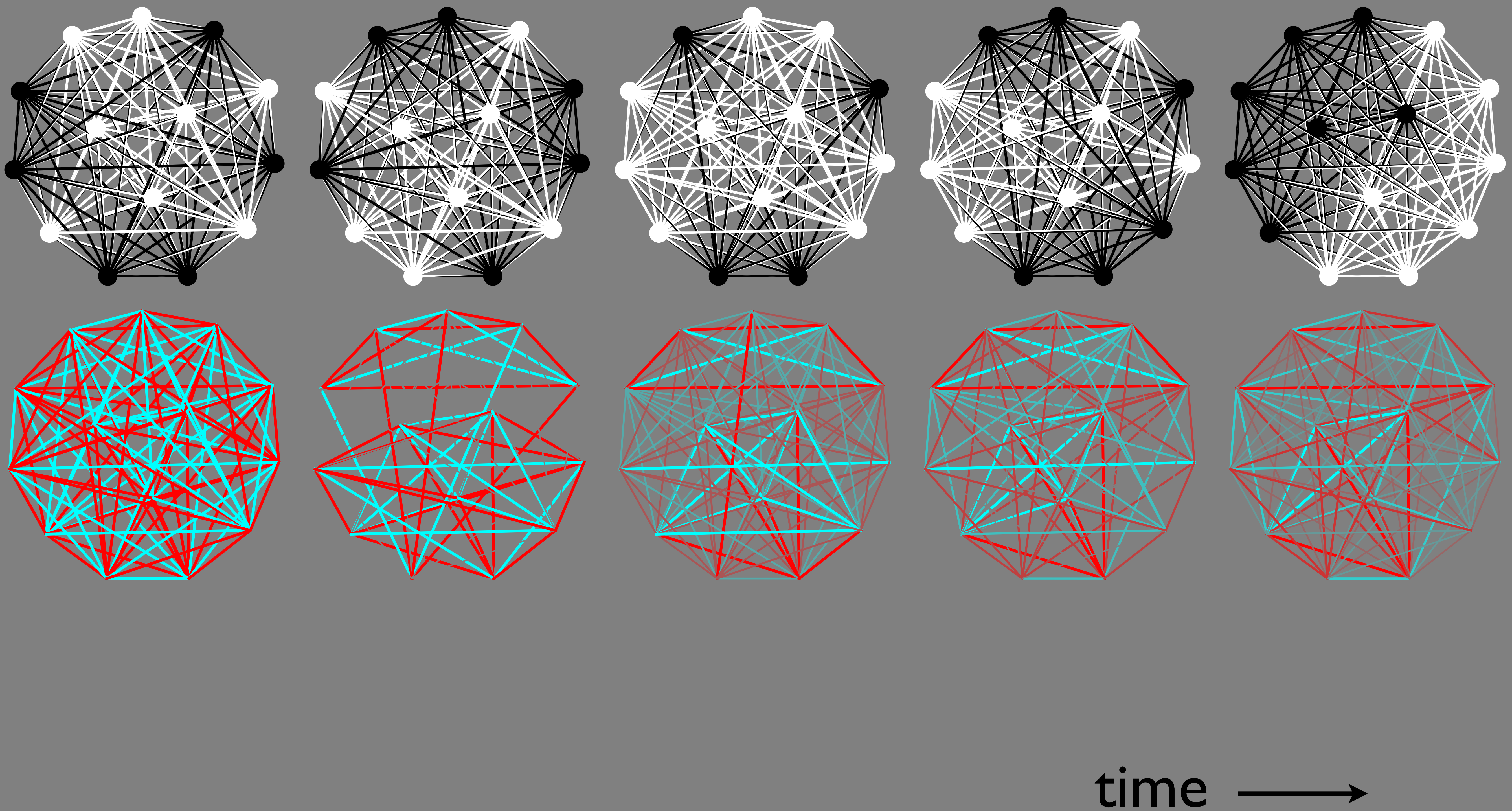


Different patterns favor different synaptic strengths



$$J_{ij} = \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$$

Synaptic weights change over time to learn all patterns

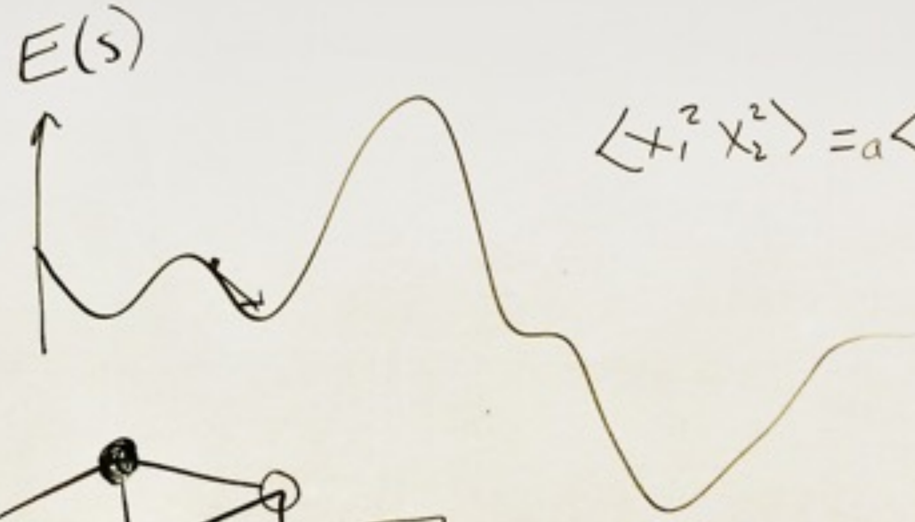

time

Demon

$$E(\underset{\sim}{s}) = -\sum_{ij} J_{ij} s_i s_j - \sum_i h_i s_i \rightarrow 0$$

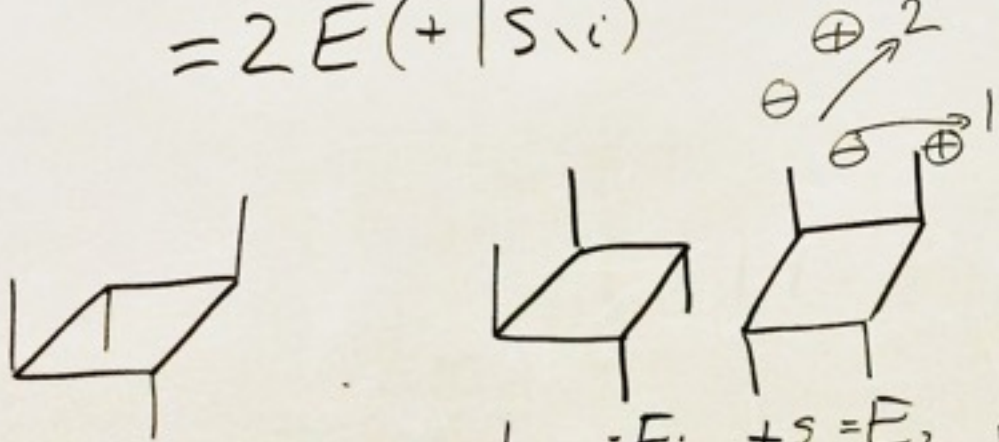
$$E(s_i \mid s_{\setminus i}) = \sum_i s_i \Big(-\sum_j J_{ij} s_j - h_i\Big)$$

$$\Delta E_i = E(+1 \mid s_{\setminus i}) - E(-1 \mid s_{\setminus i})$$
$$= 2E(+ \mid s_{\setminus i})$$

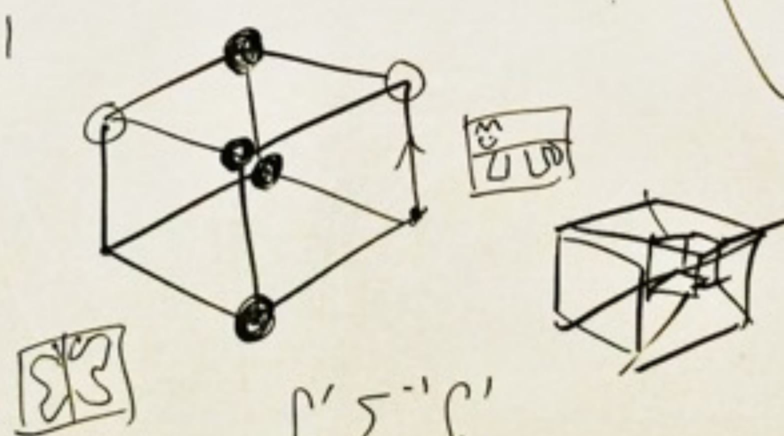


$$\langle x_1^2 x_2^2 \rangle = a\langle x_1 x_2 \rangle^2 + b\langle x_1^2 \rangle \langle x_2^2 \rangle$$

$$\longrightarrow s \in \{-1, +1\}^N$$



$$E_{ij} = -J_{ij} s_i s_j \qquad h_i s_i = E_i \qquad + s_2 = E_2$$



$$f' \Sigma^{-1} f'$$

$$\mathcal{J}_{tor} = \langle r^2 \rangle' \quad (1+d_r)^{-1} \langle r^2 \rangle$$

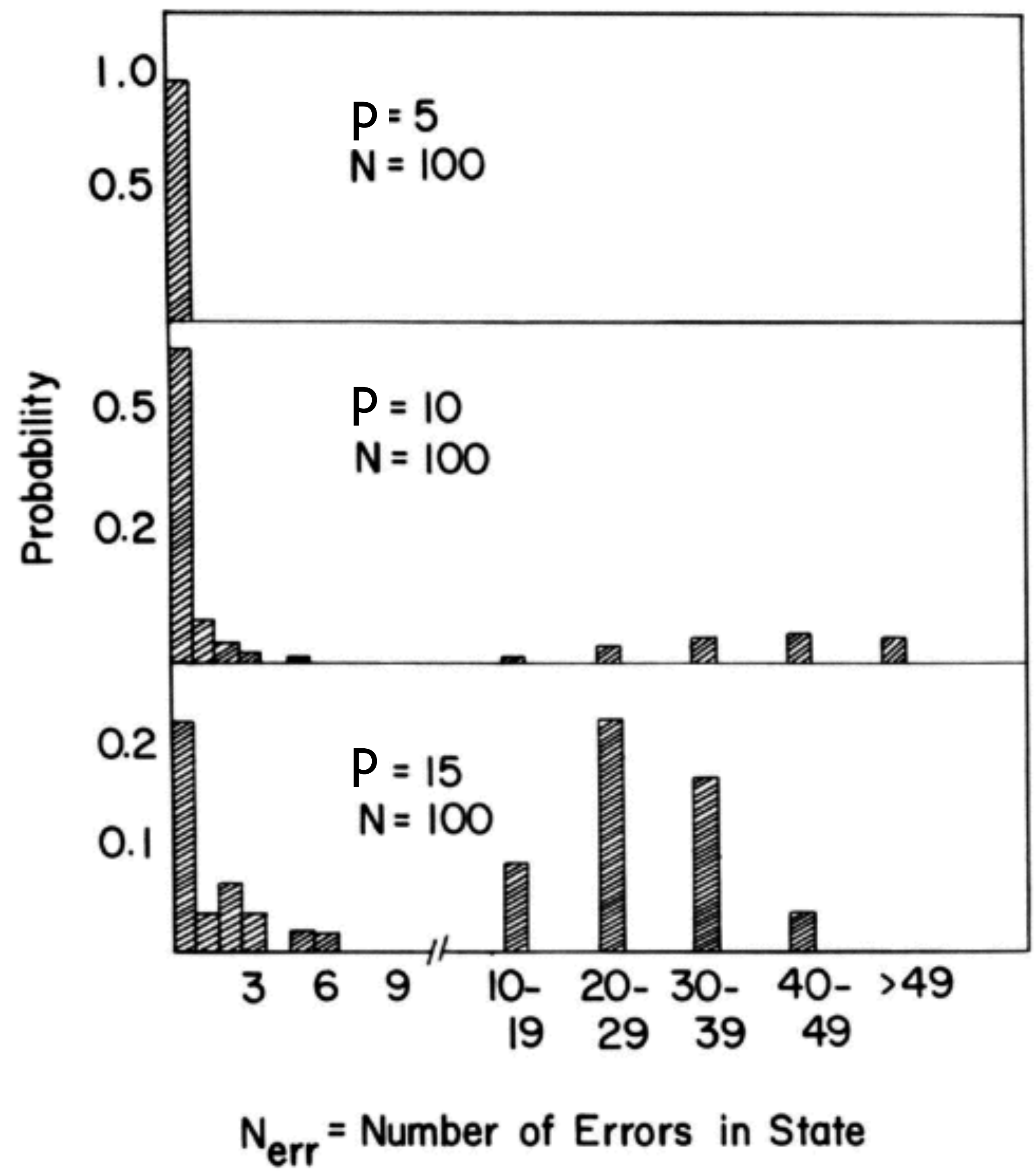
Hopfield Network Capacity

N neurons

$\sim N^2$ synapses

$\sim 2^N$ possible patterns

How many can be stored?



Hopfield 1982

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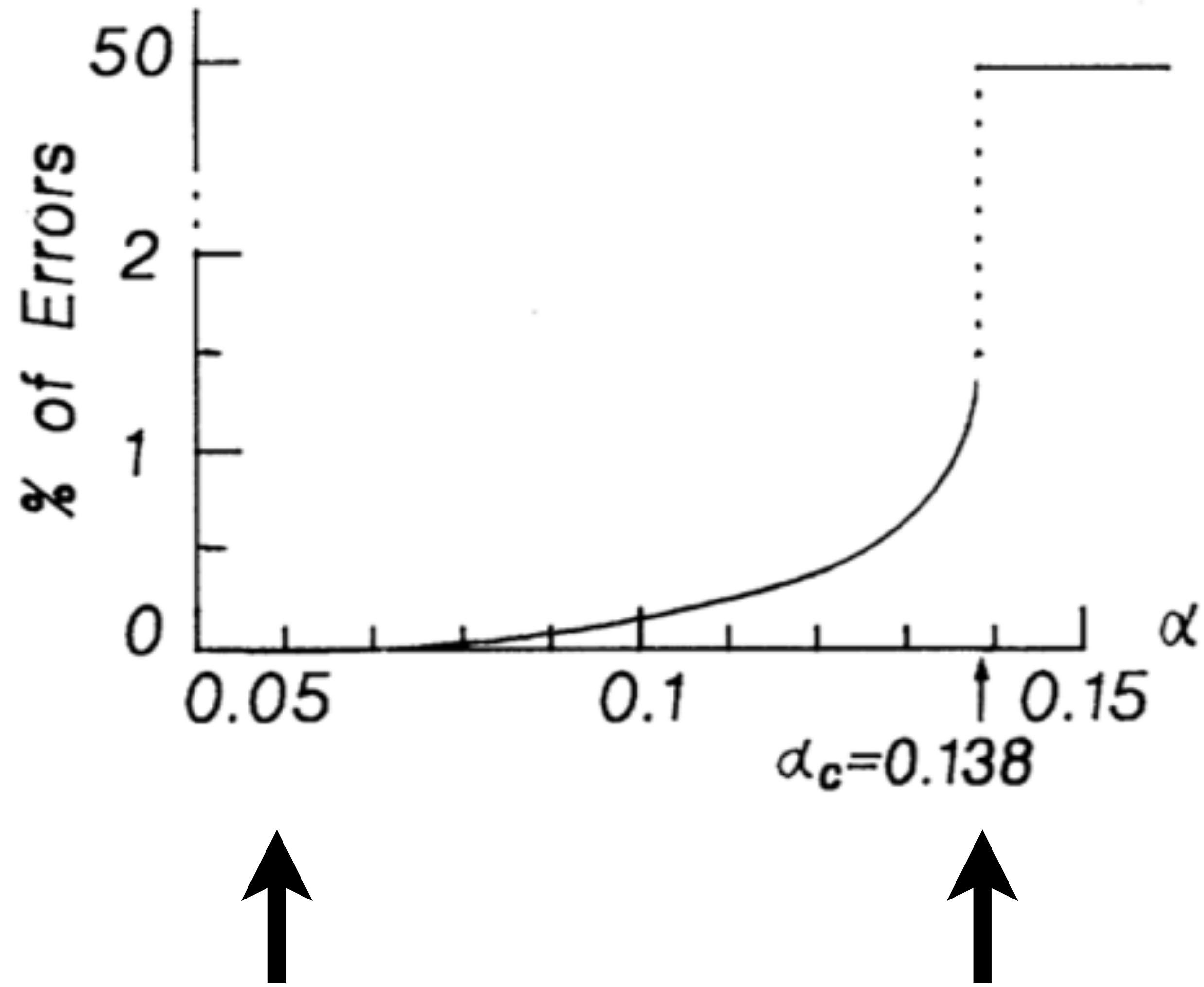

Error-free at $\sim 0.05\,N$

Capacity $\sim 0.138\,N$

Amit, Gutfreund, Sompolinsky 1985

Let’s list some of the crucial assumptions underlying the Hopfield network!

Are they realistic? Are they necessary?

Generalizations of the Hopfield net

Sparse (either inputs or connections)

Palimpsest (forgetting)

Different learning rules

Asymmetric connectivities